Winslow Township School District

Mathematics Curriculum – Algebra 2

Unit 2

Overview	Standards for Mathematical Content		Unit Focus	Standards for Mathematical Practice
Unit 2 **Polynomials and Analysis of Nonlinear Functions**	• A.APR.B.2 • A.SSE.A.2 • A.APR.B.3 • F.IF.C.7 • A.APR.C.4 • A.APR.D.6 • A.REI.A.1	• A.REI.A.2 • A.CED.A.1 • F.IF.B.4 • F.IF.B.6 • G.GPE.A.2 • F.IF.C.7 • A.REI.D.11	• Understand the relationship between zeros and factors of polynomials • Interpret the structure of expressions • Use polynomial identities to solve problems • Analyze functions using different representations • Rewrite rational expressions • Understand solving equations as a process of reasoning and explain the reasoning • Interpret functions in terms of the context • Translate between the geometric description and the equation for a conic section • Represent and solve equations and inequalities graphically	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments & critique the reasoning of others. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.
Unit 2: ***Suggested Open Educational Resources***	A.APR.B.2 The Missing Coefficient A.SSE.A.2 A Cubic Identity A.APR.B.3 Graphing from Factors III F.IF.C.7c Graphs of Power Functions A.APR.C.4 Trina's Triangles A.APR.D.6 Combined Fuel Efficiency A.REI.A.1 Products and Reciprocals		A.REI.A.2 Radical Equations A.REI.A.2, A.CED.A.1 An Extraneous Solution G.GPE.A.2 Defining Parabolas Geometrically F.IF.C.7e Logistic Growth Model A.REI.D.11 Ideal Gas Law	

Major Supporting Additional (Identified by PARCC Model Content Frameworks).

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Curriculum Unit 2	Standards		Pacing	
			Days	Unit Days
Unit 2 **Polynomials and Analysis of Nonlinear Functions**	• G.GPE.A.2 • A.APR.C.4	Derive the equation of a parabola given a focus and directrix. Use polynomial identities to describe numerical relationships and prove polynomial identities.	9	45
	• F.IF.C.7 • A.APR.D.6 • A.CED.A.1	Graph polynomial functions from equations; identify zeros when suitable factorizations are available; show key features and end behavior. Rewrite simple rational expressions in different forms using inspection, long division, or, for the more complicated examples, a computer algebra system. Create equations and inequalities in one variable and use them to solve problems.	13	
	• A.APR.B.2 • A.SSE.A.2 • A.APR.B.3 • A.REI.A.1 • A.REI.A.2 • A.REI.D.11 • F.IF.B.4 • F.IF.B.6	Apply the Remainder Theorem in order to determine the factors of a polynomial. Use an appropriate factoring technique to factor polynomials. Explain the relationship between zeros and factors of polynomials, and use the zeros to construct a rough graph of the function defined by the polynomial. Solve simple rational and radical equations in one variable, use them to solve problems and show how extraneous solutions may arise. Find approximate solutions for f(x)=g(x), using technology to graph, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, logarithmic and exponential functions. For radical functions, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	18	
	Assessment, Re-teach and Extension		5	

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Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**
• A.APR.B.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.	MP.6 Attend to precision.	Concepts: • Polynomial division: For a polynomial $p(x)$ and a number a: – $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$ – $(x – a)$ is a factor of $p(x)$ if and only if $p(a) = 0$ Students are able to: • use the Remainder Theorem to determine factors of a polynomial. Learning Goal 1: Apply the Remainder Theorem in order to determine the factors of a polynomial.
• A.SSE.A.2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 – y^4$ *as* $(x^2)^2 – (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 – y^2)(x^2 + y^2)$. • A.APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	MP.7 Look for and make use of structure.	Concepts: • Factors of polynomials can be used to identify zeros to be used to develop a rough graph of the polynomial function. Students are able to: • factor polynomials. • analyze a table of values to determine where the polynomial is increasing and decreasing. • use the zeros of the polynomial to create rough graph. Learning Goal 2: Use an appropriate factoring technique to factor polynomials. Explain the relationship between zeros and factors of polynomials, and use the zeros to construct a rough graph of the function defined by the polynomial.
• F.IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.C.7c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	MP.1 Make sense of problems and persevere in solving them. MP.5 Use appropriate tools strategically. MP.6 Attend to precision.	Concepts: • Factors of polynomials can be used to identify zeros to be used to develop a rough graph of the polynomial function. Students are able to: • graph a polynomial function given its equation. • identify zeros from the graph and using an appropriate factoring technique. • show key features of the graph, including end behavior. • use technology to graph and describe key features of the graph for complicated cases. Learning Goal 3: Graph polynomial functions from equations; identify zeros when suitable factorizations are available; show key features and end behavior.

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• A.APR.C.4. Prove polynomial identities and use them to describe numerical relationships. *For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity* $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	MP.3 Construct viable arguments and critique the reasoning of others. MP.7 Look for and make use of structure.	Concepts: • Polynomial identities can be used to describe numerical relationships. Students are able to: • show that the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. • prove polynomial identities. Learning Goal 4: Use polynomial identities to describe numerical relationships and prove polynomial identities.
• A.APR.D.6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	MP.1 Make sense of problems and persevere in solving them.	Concepts: • Rational expressions can be written in different forms. Students are able to: • write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$. • use inspection, factoring and long division to rewrite rational expressions. • use technology to rewrite rational expressions for more complicated cases. Learning Goal 5: Rewrite simple rational expressions in different forms using inspection, long division, or, for the more complicated examples, a computer algebra system.
• A.REI.A.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. • A.REI.A.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. • A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MP.6 Attend to precision.	Concepts: • Inverse relationships exist between roots and powers. • Extraneous solutions do not result in true statements. Students are able to: • use the inverse relationship between roots and powers when solving radical equations. • identify any extraneous solutions. • solve simple rational equations in one variable (degree of numerators and denominator is not greater than 2). • write simple rational equations in one variable and use the rational equation to solve problems. Learning Goal 6: Solve simple rational and radical equations in one variable, use them to solve problems and show how extraneous solutions may arise. Create simple rational equations in one variable and use them to solve problems.

• F.IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* • F.IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	MP.1 Make sense of problems and persevere in solving them. MP 4. Model with mathematics MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure.	Concepts: • A radical function is any function that contains a variable inside a root. Students are able to: • interpret key features of radical functions from graphs and tables in the context of the problem. • sketch graphs of radical functions given a verbal description of the relationship between the quantities. • identify intercepts and intervals where function is increasing/decreasing. • determine the practical domain of a radical function. • determine key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior. Learning Goal 7: For radical functions, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
• G.GPE.A.2. Derive the equation of a parabola given a focus and directrix	MP.1 Make sense of problems and persevere in solving them. MP.5 Use appropriate tools strategically.	Concepts: • Any point on a parabola is equidistant between the focus and the directrix. Students are able to: • use the distance formula to write an equation of a parabola when the focus and directrix are given. Learning Goal 8: Derive the equation of a parabola given a focus and directrix.
• F.IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.C.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	MP.1 Make sense of problems and persevere in solving them. MP 4. Model with mathematics	Concepts: • Logarithmic functions Students are able to: • graph logarithmic functions having base 2, 10 or e, using technology for more complicated cases. • show intercepts and end behavior of logarithmic functions. Learning Goal 9: Graph logarithmic functions expressed symbolically and show key features of the graph (including intercepts and end behavior).
• A.REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to	MP.1 Make sense of problems and persevere in solving them.	Concepts: • Solutions to complex systems of nonlinear functions can be approximated graphically Students are able to: • find the solution to f(x)=g(x) approximately, e.g., using technology to graph the functions; include cases where f(x) and/or g(x) are linear,

graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	MP.5 Use appropriate tools strategically.	polynomial, rational, absolute value, exponential, and logarithmic functions. ● find the solution to f(x)=g(x) approximately, e.g., using technology to make tables of values, or find successive approximations; include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Learning Goal 10: Find approximate solutions for f(x)=g(x), using technology to graph, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, logarithmic and exponential functions.

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District/School Formative Assessment Plan	**District/School Summative Assessment Plan**
Pre-Assessment, Quizzes Exit Tickets Daily Monitoring Linkit!	Unit Benchmark Linkit! PARCC Diagnostic

Focus Mathematical Concepts

Prerequisite skills:

Students should be able to:

- determine the number of monomials in each expression
- use distributive property to write expressions as an equivalent algebraic expression
- determine whether an equation is linear
- understand that linear functions can be represented in different ways

Common Misconceptions:

Students may believe that the use of algebraic expressions is merely the abstract manipulation of symbols. Use of real- world context examples to demonstrate the meaning of the parts of algebraic expressions is needed to counter this misconception.

Students may also believe that an expression cannot be factored because it does not fit into a form they recognize. They need help with reorganizing the terms until structures become evident.

Students will often combine terms that are not like terms. For example, $2 + 3x = 5x$ or $3x + 2y = 5xy$.

Students sometimes forget the coefficient of 1 when adding like terms. For example, $x + 2x + 3x = 5x$ rather than $6x$.

Students will change the degree of the variable when adding/subtracting like terms. For example, $2x + 3x = 5x^2$ rather than $5x$.

Students will forget to distribute to all terms when multiplying. For example, $6(2x + 1) = 12x + 1$ rather than $12x + 6$.

Students may not follow the Order of Operations when simplifying expressions. For example, $4x^2$ when $x = 3$ may be incorrectly evaluated as $4 \cdot 3^2 = 12^2 = 144$, rather than $4 \cdot 9 = 36$. Another common mistake occurs when the distributive property should be used prior to adding/subtracting. For example, $2 + 3(x - 1)$ incorrectly becomes $5(x - 1) = 5x - 5$ instead of $2 + 3(x - 1) = 2 + 3x - 3 = 3x - 1$.

Students may believe that the graph of a function is simply a line or curve "connecting the dots," without recognizing that the graph represents all solutions to the equation.

Students may also believe that graphing linear and other functions is an isolated skill, not realizing that multiple graphs can be drawn to solve equations involving those functions.

Additionally, students may believe that two-variable inequalities have no application in the real world. Teachers can consider business related problems (e.g., linear programming applications) to engage students in discussions of how the inequalities are derived and how the feasible set includes all the points that satisfy the conditions stated in the inequalities.

Fluency Recommendations:

A-SSE.A.2 The ability to see structure in expressions and to use this structure to rewrite expressions is a key skill in everything from advanced factoring (e.g., grouping) to summing series to the rewriting of rational expressions to examine the end behavior of the corresponding rational function.

F-IF.A.3 Fluency in translating between recursive definitions and closed forms is helpful when dealing with many problems involving sequences and series, with applications ranging from fitting functions to tables to problems in finance.

District/School Tasks	District/School Primary and Supplementary Resources and Technology Integration
PARCC Released Items **http://www.parcc-assessment.org/released-items** **NJDOE Digital Item Library** **https://nj.digitalitemlibrary.com/home** **NJSLA Mathematics Evidence Statements** **https://docs.google.com/spreadsheets/d/18M5r1jk4P729fTpAlWAzrw1gE6tken233I-Yk0U712M/edit#gid=554025491** **LinkIt! Form A, B, & C**	**Textbook** **IXL** https://www.ixl.com/ **Khan Academy** https://www.khanacademy.org/ **HS Flip Book:** http://community.ksde.org/Default.aspx?tabid=5646 **North Carolina Dept of Ed. Wikispaces:** http://maccss.ncdpi.wikispaces.net/High+School **101 Math Discourse Questions:** http://www.casamples.com/downloads/100MathDiscourseQuestions_Printable.pdf **Asking Effective Questions** http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/CBS_AskingEffectiveQuestions.pdf **Diversity, Equity & Inclusion Educational Resources** https://www.nj.gov/education/standards/dei/

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Instructional Best Practices and Exemplars	
1. Identifying similarities and differences 2. Summarizing and note taking 3. Reinforcing effort and providing recognition 4. Homework and practice 5. Nonlinguistic representations	6. Cooperative learning 7. Setting objectives and providing feedback 8. Generating and testing hypotheses 9. Cues, questions, and advance organizers 10. Manage response rate

Vocabulary			
Binomial Theorem coefficient exponential factors	factorization finite function Geometric Series	infinite logarithmic polynomial	relation terms zeros

9.1 Personal Financial Literacy, 9.2 Career Awareness, Exploration, Preparation and Training & 9.4 Life Literacies and Key Skills

9.1.12.CDM.6: Compute and assess the accumulating effect of interest paid over time when using a variety of sources of credit. (e.g., student loans, credit cards, auto loans, mortgages, etc.).
9.1.12.CDM.7: Calculate a mortgage payment based on type of loan, down payment, credit score, and loan interest rate.
9.1.12.CP.3: Summarize factors that affect a positive credit rating, including on-time payments, debt versus available credit, length of open credit, and how often you apply for credit.
9.1.12.CP.5: Create a plan to improve and maintain an excellent credit rating.
9.1.12.EG.1: Review the tax rates on different sources of income and on different types of products and services purchased.
9.1.12.EG.2: Explain why various forms of income are taxed differently.
9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.4: Assess and critique the appropriateness and impact of existing data visualizations for an intended audience (e.g., S-ID.B.6b, HS-LS2-4).
9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments.
The implementation of the 21st Century skills and standards for students of the Winslow Township District is infused in an interdisciplinary format in a variety of curriculum areas that include, English language Arts, Mathematics, School Guidance, Social Studies, Technology, Visual and Performing Arts, Science, Physical Education and Health, and World Language.
Additional opportunities to address 9.1, 9.2 & 9.4:
Philadelphia Mint
https://www.usmint.gov/learn/kids/resources/educational-standards
Different ways to teach Financial Literacy.
https://www.makeuseof.com/tag/10-interactive-financial-websites-teach-kids-money-management-skills/

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Suggested Modifications for Special Education/504
Students with special needs: The students' needs will be addressed on an individual and grade level using a variety of modalities. Accommodations will be made for those students who need extra time to complete assignments. Support staff will be available to aid students related to IEP specifications. 504 accommodations will also be attended to by all instructional leaders. Physical expectations and modifications, alternative assessments, and scaffolding strategies will be used to support this learning. The use of Universal Design for Learning (UDL) will be considered for all students as teaching strategies are considered.
☐ Provide the opportunity to re-take tests ☐Modify activities/assignments/projects/assessments ☐ Breakdown activities/assignments/projects/assessments into manageable units ☐Additional time to complete activities/assignments/projects/assessments ☐ Provide an option for alternative activities/assignments/projects/assessments ☐ Modify Content ☐ Modify Amount ☐ Small Group Intervention/Remediation ☐ Individual Intervention/Remediation ☐ Additional Support Materials ☐ Guided Notes ☐ Graphic Organizers ☐ Adjust Pacing of Content ☐ Increase one on one time ☐ Peer Support ☐ Other Modifications for Special Education:
Suggested Modifications for At-Risk Students
Formative and summative data will be used to monitor student success. At first signs of failure, student work will be reviewed to determine support. This may include parent consultation, basic skills review and differentiation strategies. With considerations to UDL, time may be a factor in overcoming developmental considerations
☐ Provide the opportunity to re-take tests ☐ Increase one on one time ☐ Oral prompts can be given ☐ Using visual demonstrations, illustrations, and models ☐ Give directions/instructions verbally and in simple written format ☐ Peer Support ☐ Modify activities/assignments/projects/assessments ☐ Additional time to complete activities/assignments/projects/assessments ☐ Provide an option for alternative activities/assignments/projects/assessments ☐ Modify Content ☐ Modify Amount ☐ Adjust Pacing of Content ☐ Small Group Intervention/Remediation ☐ Individual Intervention/Remediation ☐ Additional Support Materials ☐ Guided Notes ☐ Graphic Organizers ☐ Other Modifications for Students At-Risk:

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Suggested for English Language Learners	Suggested Modifications for Gifted Students
All WIDA Can Do Descriptors can be found at this link: https://wida.wisc.edu/teach/can-do/descriptors ☐ Grades 9-12 WIDA Can Do Descriptors: ☐ Listening ☐ Speaking ☐ Reading ☐ Writing ☐ Oral Language Students will be provided with accommodations and modifications that may include: • Relate to and identify commonalities in mathematics studies in student's home country • Assist with organization • Use of computer • Emphasize/highlight key concepts • Teacher Modeling • Peer Modeling • Label Classroom Materials - Word Walls	Students excelling in mastery of standards will be challenged with complex, high level challenges related to the topic. • Raise levels of intellectual demands • Require higher order thinking, communication, and leadership skills • Differentiate content, process, or product according to student's readiness, interests, and/or learning styles • Provide higher level texts • Expand use of open-ended, abstract questions • Critical and creative thinking activities that provide an emphasis on research and in-depth study • Enrichment Activities/Project-Based Learning/ Independent Study Additional Strategies may be located at the links: ❖ Gifted Programming Standards ❖ Webb's Depth of Knowledge Levels and/or Revised Bloom's Taxonomy ❖ REVISED Bloom's Taxonomy Action Verbs
Suggested Activities	
☐ Do Now/Warm-Up ☐ Whole Group ☐ Small Groups ☐ Guided Practice ☐ Independent Practice	☐ Centers ☐ Intervention/Remediation ☐ Projects ☐Academic Games ☐ Other Suggested Activities:

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Interdisciplinary Connections
Big Ideas Real-Life STEM Videos and Performance Tasks **Interdisciplinary Connections: ELA** **NJSLSA.R1**. Read closely to determine what the text says explicitly and to make logical inferences and relevant connections from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text. **NJSLSA.W2.** Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content **NJSLSA.L1**. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking **SL.9-10.4:** Present information, findings and supporting evidence clearly, concisely and logically. The content, organization, development and style are appropriate to task, purpose and audience. **NJSLSA.L6:** Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.
Integration of Computer Science and Design Thinking NJSLS 8
8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena. 8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process. 8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms. 8.1.12.AP.2: Create generalized computational solutions using collections instead of repeatedly using simple variables. 8.1.12.AP.8: Evaluate and refine computational artifacts to make them more usable and accessible. 8.2.12.ETW.2: Synthesize and analyze data collected to monitor the effects of a technological product or system on the environment. • 8.2.12.ETW.3: Identify a complex, global environmental or climate change issue, develop a systemic plan of investigation, and propose an innovative sustainable solution.